Example **A wildlife photographer is taking photographs of a rare glass frog. He's found over time that the probability that he'll sight a glass frog during any day of searching is 0.05. He moves to another part of the rainforest where he claims the probability will be different. During his first 6 days searching he spots the frog on 3 of the days. Use a 1% level of significance to test his claim.**

1. Identify the population parameter.

   Let p = probability that the wildlife photographer will spot a glass frog in a day of searching.

2. Formulate the hypotheses.

   $H_0: p = 0.05$, $H_1: p \neq 0.05$

3. State the test statistic.

   Let X = number of sampled days that he spots a frog. Under H_0, $X \sim B(6, 0.05)$

4. State the significance level.

   $\alpha = 0.01$, so $\frac{\alpha}{2} = 0.005$

5. The test is two-tailed, so you need to work out which 'tail' you are working in.
   Find the number of days on which the photographer expects to see a glass frog.

   Under H_0, the photographer believes he'll see a glass frog on 5% of days when he looks. So in 6 days, he'd expect to see a glass frog on $0.05 \times 6 = 0.3$ days.

   The observed number of days (3) is greater than this expected value under H_0, so you're in the upper tail. So find $P(X \geq 3)$.

   $$P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2)$$
   $$= 1 - 0.9977... = 0.0022 \text{ (4 d.p.)}$$

   Since $0.0022 \leq 0.005$, the result is significant.

6. Write your conclusion.

   There is sufficient evidence at the 1% level of significance to reject H_0 in favour of the wildlife photographer's claim that the probability of sighting a glass frog is different in the other part of the rainforest.

Exercise 2.3

Q1 In the past, 25% of John's violin pupils have gained distinctions in their exams. He's using a different examination board and wants to know if the percentage of distinctions will be significantly different. His first 12 exam candidates gained 6 distinctions. Test whether the percentage of distinctions is significantly different at the 1% level.

Q2 Jin is a keen birdwatcher. Over time he has found that 15% of the birds he sees are classified as 'rare'. He has bought a new type of birdseed and is not sure whether it will attract more or fewer rare birds. On the first day only 2 out of 40 of the birds were rare. Test whether the percentage of rare birds is significantly different at the 10% level.

Q3 10% of customers at a village newsagent's buy Pigeon Spotter Magazine. The owner has just opened a new shop in a different village and wants to know whether this proportion will be different in the new shop. One day 8 out of a random sample of 50 customers bought Pigeon Spotter Magazine. Is this significant at the 5% level?

Q4

When people are asked "what is your favourite day of the week?", it is thought that on average one person in four replies "Sunday".

To test this assertion 15 people were asked this question and 7 replied "Sunday".

Carry out a significance test, at the 5% level, of whether the statement "on average the preferred day of the week is Sunday, for one in four persons".

Q5

In a craft activity in a primary school, kids use beads which are kept in a bag. The bag contains a large number of beads of different colours. It is known that 0.3 of the beads are coloured gold.

The teacher claims the proportion of gold beads in the bag has changed after the activity.

She checks a random sample of 20 beads out of the bag, after the end of the activity

She finds two gold beads in the sample.

Test, at the 5% level of significance, whether or not there is evidence to support the teacher's claim.

Answers

Q1.
Let p be the proportion of pupils gaining distinctions in their exams.
$H0: p = 0.25$
$H1: p \neq 0.25$
Let X be the number of pupils in the sample gaining distinctions in their exams.
Under H0, $X \sim B(12, 0.25)$
$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9456 = 0.0544$
$\alpha/2 = 0.005$
$0.0544 > 0.005$
Not significant. Do not reject H0. There is not sufficient evidence to suggest that the proportion of pupils gaining distinctions in their exams has changed.

Q2.
Let p be the proportion of birds seen that are rare.
$H0: p = 0.15$
$H1: p \neq 0.15$
Let X be the number of birds in the sample that are rare.
Under H0, $X \sim B(40, 0.15)$
$P(X \leq 2) = 0.0486$
$\alpha/2 = 0.05$
$0.0486 < 0.05$
Significant. Reject H0. There is sufficient evidence to suggest that the proportion of birds seen that are rare is different with the new birdseed.

Q3.
Let p be the proportion of customers that buy Pigeon Spotter Magazine.
$H0: p = 0.1$
$H1: p \neq 0.1$
Let X be the number of customers in the sample that buy Pigeon Spotter Magazine.
Under H0, $X \sim B(50, 0.1)$
$P(X \geq 8) = 1 – P(X \leq 7) = 1 – 0.8779 = 0.1221$
$\alpha/2 = 0.025$
$0.1221 > 0.025$
Not significant. Do not reject H0. There is not sufficient evidence to suggest that the proportion of customers that buy Pigeon Spotter Magazine has changed.

Q4

X = NUMBER OF PEOPLE WHO FAVOUR SUNDAY

$X \sim B(15, 0.25)$

$H_0: p = 0.25$

$H_1: p \neq 0.25$, WHERE p IS THE PREFERENCE FOR SUNDAY FOR ALL PEOPLE

TESTING AT 5% (TWO TAILED) ON THE BASIS THAT $x = 7$

$P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.943379\ldots$

$= 0.0566203\ldots$

$= 5.66\%$

$> 2.5\%$ ← TWO TAILED AT 5%

THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE VALIDITY OF THE STATEMENT

INSUFFICIENT EVIDENCE TO REJECT H_0

Q5

X = NUMBER OF GOLD BEADS

$X \sim B(20, 0.3)$

$H_0: p = 0.3$

$H_1: p \neq 0.3$

WHERE p IS THE PROPORTION OF GOLD BEADS IN GENERAL

TESTING AT 5% SIGNIFICANCE (2.5% IN EACH TAIL), ON THE BASIS THAT $x = 2$

$P(X \leq 2) = 0.03548\ldots$

$= 3.55\%$

$> 2.5\%$

THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE TEACHER'S CLAIM

THERE IS NO SUFFICIENT EVIDENCE TO REJECT H_0